

Exercise 57

Find a formula for a function f that satisfies the following conditions:

$$\lim_{x \rightarrow \pm\infty} f(x) = 0, \quad \lim_{x \rightarrow 0} f(x) = -\infty, \quad f(2) = 0,$$

$$\lim_{x \rightarrow 3^-} f(x) = \infty, \quad \lim_{x \rightarrow 3^+} f(x) = -\infty$$

Solution

To make $f(2) = 0$, put $x - 2$ in the numerator. To make $\lim_{x \rightarrow \pm\infty} f(x) = 0$, make sure the degree of the denominator is higher than that of the numerator. To make $\lim_{x \rightarrow 0} f(x) = -\infty$, put x^2 in the denominator. To make $\lim_{x \rightarrow 3^\pm} f(x) = \mp\infty$, place $3 - x$ in the denominator.

$$f(x) = \frac{x - 2}{x^2(3 - x)}$$

